Slot-Coupled Tee Junction in Rectangular-Guide *E* Plane

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Abstract—**An expression for the equivalent circuit of a waveguide tee junction coupled through a narrow transverse slot of variable length is determined from self-reaction and discontinuity in modal voltage. Impedance loading on primary guide and coupling are determined from the equivalent network parameter. A comparison between theoretical and experimental results is presented.**

I. Introduction

ELECTROMAGNETIC coupling between two rectangular waveguides forming a tee junction coupled through narrow transverse slot in broad wall of primary waveguide can be determined from a knowledge of equivalent circuit of the aperture. The determination of equivalent circuit parameter from polarizabilities of the aperture using Bethe's [1] formulation based on asymptotic representation by dipole moments is limited to slots having length small compared with the wavelength. Bethe's theory was extended to the case of long slots by Cohn [2] using experimental results on polarizability [3]. Marcuvitz [4] and Levinson and Fredberg [5], [6] have obtained an expression for the network parameter only for the particular case of a slot having length equal to the broad dimension of the primary waveguide. Levinson and Fredberg have not, however, suggested any method of obtaining closed-form expression for the general case of a slot of any arbitrary length from the solution of integral equation formulated by them. A knowledge of the variation of equivalent circuit parameter as a function of the length of the transverse slot enables one to determine the variation of not only the coupling but also the real and imaginary parts of impedance loading on the primary guide. The later information is useful for the analysis of cascaded sections of such junction using loaded-line analysis [7].

In the present paper a variational expression for the equivalent circuit of a tee junction coupled through a transverse slot in the broad wall of primary waveguide is derived in terms of self-reaction [8] and discontinuity in modal voltage [9]. A closed-form expression for the equivalent network parameter is derived for the case of a thin rectangular transverse slot of variable length in the broad wall of the rectangular waveguide. Comparison of computed results with those obtained from Cohn's theory is presented. Coupling and impedance loading on primary waveguide due to the aperture is computed from the equivalent circuit parameter. The computed results show a good agreement with the experimental results. For a slot of length equal to the broad dimension of the rectangular guide ($2l = a$) results are compared with those calculated from Marcuvitz's and Levinson and Fredberg's formula.

Manuscript received November 22, 1977; revised February 28, 1978.
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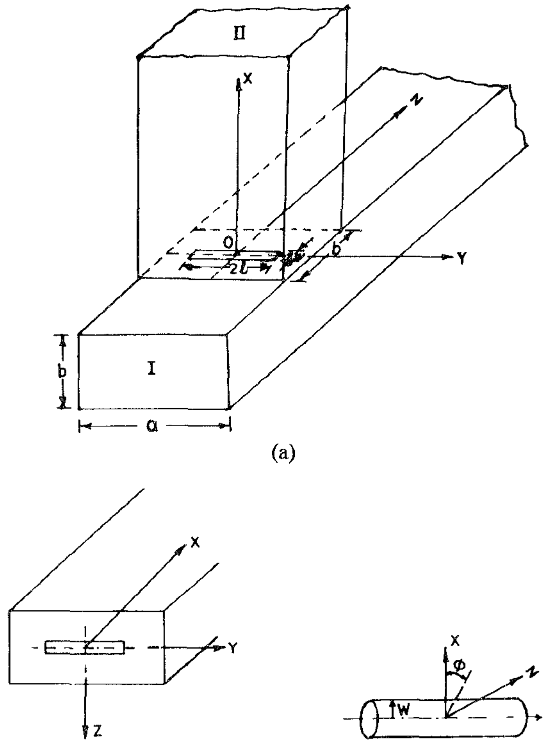

Fig. 1. (a) Tee junction coupled through transverse slot in broad wall of primary guide. (b) Auxiliary (coupled) guide with coordinate frame. (c) Equivalent cylindrical dipole and coordinate geometry.

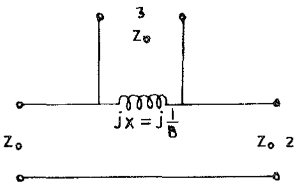

Fig. 2. Equivalent circuit of the tee junction at reference plane XOY.

II. Determination of the Equivalent Circuit Parameter

Fig. 1(a) shows a waveguide tee junction, the mechanism of coupling being a transverse slot of length $2l$ and width $2w$ in the broad wall of the guide I. Equivalent circuit parameter shown in Fig. 2 is given by

$$B=\frac{\langle a,a\rangle}{V^2} \tag{1}$$

where $\langle a,a\rangle$ is the self-reaction due to an assumed field distribution in the slot and V is the discontinuity in the modal voltage for dominant mode wave in guide I.

The self-reaction appearing in (1) is given by

$$\langle a,a\rangle=-\int_v \bar{H}_y\cdot\bar{J}_y\,dv \tag{2}$$

where $\bar{H}_y$ is the magnetic field inside the waveguide due to the voltage on the slot and $\bar{J}_y$ is the total equivalent magnetic current. Since part of the coupled volume v is in the guide I and the other part in guide II, the self-reaction $\langle a,a\rangle$ is equal to the sum of self-reactions $\langle a,a\rangle_1$ and $\langle a,a\rangle_2$ in the two volumes

$$\langle a,a\rangle_1=-\int_{v_1} \bar{H}_{y_1}\cdot\bar{J}_y\,dv \tag{3a}$$

where $\bar{H}_{y_1}$ is the magnetic field inside the primary waveguide I due to the voltage on the slot and

$$\langle a,a\rangle_2=-\int_{v_2} \bar{H}_{y_2}\cdot\bar{J}_y\,dv \tag{3b}$$

$\bar{H}_{y_2}$ being the magnetic field inside guide II.

The magnetic fields $\bar{H}_{y_1}$ and $\bar{H}_{y_2}$ in guides I and II, respectively, result from the magnetic current $\bar{J}_y$ in the aperture plane of the slot. $\bar{J}_y$ is related to the electric field in the aperture plane through the relation

$$\bar{J}_y=\bar{E}\times\bar{n} \tag{4}$$

where $\bar{n}$ is the unit normal which is specified in terms of coordinate frame.

The electric field in guide II (shown in Fig. 1(b) separately) is produced due to the field distribution of the form

$$\bar{E}=\bar{u}_z E_0\sin k(l-|y|),\qquad \text{for}\begin{cases} x=0\\ -l<y<+l\\ -w<z<+w\end{cases}$$
$$=0,\qquad \text{elsewhere} \tag{5}$$

where

$$k=\frac{2\pi}{\lambda}.$$

The electric field in the plane $x=0$ of Fig. 1(b) can be expanded in terms of the modes transverse electric to y as follows:

$$E_{z_{nm}}=\sum_n\sum_m A_{nm}\sin\frac{m\pi}{a}\left(y+\frac{a}{2}\right)\cos\frac{n\pi}{b}\left(z+\frac{b}{2}\right). \tag{6}$$

From (5) and (6) it can be shown that

$$A_{nm}=\sum_m\frac{2\epsilon_n V_0 k}{ab\left\{k^2-\left(\frac{m\pi}{a}\right)^2\right\}}\sin\frac{m\pi}{2}\left(\cos\frac{m\pi l}{a}-\cos kl\right) \tag{7}$$

where

$$V_0=2w\cdot E_0.$$

The magnetic field H_{y_2} in guide II is given by (6) and (7)

$$H_{y2}=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{2\epsilon_n V_0 k}{j\omega\mu\pi ab\gamma_{nm}}\sin\frac{m\pi}{2}\left(\cos\frac{m\pi l}{a}-\cos kl\right)\cdot\sin\frac{m\pi}{a}\left(y+\frac{a}{2}\right)\cos\frac{n\pi}{b}\left(z+\frac{b}{2}\right)e^{-\gamma_{nm}x} \tag{8}$$

where $\epsilon_n=1$ when $n=0$ and $\epsilon_n=2$ when $n=1,2,3,\cdots$.

The propagation constant

$$\gamma_{nm}=\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2-k^2}\,. \tag{9}$$

The magnetic field H_{y_1} in guide I is related to the vector potential through the expression

$$H_{y_1}=\frac{1}{j\omega\mu}\left[k^2A+\frac{\partial^2 A}{\partial y^2}\right]. \tag{10}$$

The general expression for the vector potential due to y-directed current in a waveguide has been derived by Markov [10]. Use of the same expression in (10) leads to H_{y_1} in the following form:

$$H_{y_1}=\frac{1}{j\omega\mu}\left[\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\left\{k^2-\left(\frac{m\pi}{a}\right)^2\right\}\frac{\epsilon_n}{ab\gamma_{nm}}\cdot\cos\frac{n\pi}{b}(x+b)\sin\frac{m\pi}{a}\left(y+\frac{a}{2}\right)\cdot\int_s\left\{\cos\frac{n\pi}{b}(x'+b)\sin\frac{m\pi}{a}\left(y'+\frac{a}{2}\right)\cdot\left(e^{-\gamma_{nm}z}\int_{-\infty}^{z'=z}\bar{J}_y^s(x',y',z')e^{\gamma_{nm}z'}dz'+e^{\gamma_{nm}z}\cdot\int_{z'=z}^{\infty}\bar{J}_y^s(x',y',z')e^{-\gamma_{nm}z'}dz'\right)\right\}ds\right] \tag{11}$$

where the index S of the sign of the integral indicates that the integration is performed over the cross section of the waveguide and $\bar{J}_y^s$ is the surface current density. For a very narrow slot, the variation along z' axis may be approximated by delta function. Hence, from (4), (5), and (11) the magnetic field due to the transverse slot is obtained as

$$H_{y_1}=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{2V_0\epsilon_n}{j\omega\mu\pi ab\gamma_{nm}}\left\{k^2-\left(\frac{m\pi}{a}\right)^2\right\}\cdot\sin\frac{m\pi}{a}\left(y+\frac{a}{2}\right)\cos\frac{n\pi}{b}(x+b)\cdot\left[\frac{\sin\frac{m\pi}{2}\left(\cos\frac{m\pi l}{a}-\cos kl\right)}{k\left\{1-\left(\frac{m\lambda}{2a}\right)^2\right\}}\right]e^{-\gamma_{nm}z}. \tag{12}$$

For convenient evaluation of the volume integral appearing in (3a) and (3b), the slot is replaced by an equivalent cylindrical dipole having diameter equal to the slot width as shown in Fig. 1(c) with the coordinate system.

Since

$$x+b=w\cos\phi$$
$$z=w\sin\phi.$$

The expression for self reaction $\langle a,a\rangle_1$ is obtained from (3a), (4), (5), and (12) as follows:

$$\langle a,a\rangle_1=\sum_{n=0}^{\infty}\sum_{m=1,3,5,\cdots}^{\infty}\frac{2jV_0^2\epsilon_n}{\pi^2ab\omega\mu\gamma_{nm}}\left[\frac{\left(\cos\frac{m\pi l}{a}-\cos kl\right)^2}{1-\left(\frac{m\lambda}{2a}\right)^2}\right]\cdot\int_0^{\pi}\cos\left(\frac{n\pi}{b}w\cos\phi\right)e^{-\gamma_{nm}w\sin\phi}\,d\phi. \quad (13)$$

Self-reaction is computed for $m=1$ and $m>1$ separately.

For $m=1$

$$\langle a,a\rangle_1=\frac{4jV_0^2}{\pi^2a\omega\mu}\frac{\left(\cos\frac{\pi l}{a}-\cos kl\right)^2}{\left\{1-\left(\frac{\lambda}{2a}\right)^2\right\}}\cdot\left[-\ln 2\sin\frac{\pi w}{2b}+\frac{\beta^2b^2}{2\pi^2}\sum_1^{\infty}\frac{1}{n^3}\right] \quad (14a)$$

where $\beta^2=k^2-(\pi/a)^2$.

For $m>1$, Poisson's summation formula [11] is used to convert the sum into rapidly convergent series as follows:

$$\langle a,a\rangle_1=\sum_{m=3,5,7,\cdots}^{\infty}\frac{4jV_0^2}{\pi^2a\omega\mu}\frac{\left(\cos\frac{m\pi l}{a}-\cos kl\right)}{\left\{1-\left(\frac{m\lambda}{2a}\right)^2\right\}}k_0(k_mw) \quad (14b)$$

where

$$k_m^2=\left(\frac{m\pi}{a}\right)^2-k^2$$

and k_0 is the modified Bessel function of the second kind which decays very rapidly so that the only significant terms are those for $n=0$. Total reaction $\langle a,a\rangle_1$ is given by the sum of (14a) and (14b).

Similarly, from (3b), (4), (5), and (8), the self-reaction for guide II

$$\langle a,a\rangle_2=\sum_{n=1}^{\infty}\sum_{m=1,3,5,\cdots}^{\infty}\frac{2jV_0^2}{\omega\mu\pi ab\gamma_{nm}}\frac{\left(\cos\frac{m\pi l}{a}-\cos kl\right)^2}{\left\{1-\left(\frac{m\lambda}{2a}\right)^2\right\}}\cdot\int_{\pi}^{2\pi}\cos\left(\frac{n\pi}{b}w\sin\phi+\frac{b}{2}\right)e^{-\gamma_{nm}w\cos\varphi}\,d\varphi. \quad (15)$$

Since

$$x=w\cos\phi$$
$$z=w\sin\phi.$$

Expression (15) is evaluated for $m=1$ and $m>1$ separately as earlier. The results are as follows:

for $m=1$

$$\langle a,a\rangle_2=\frac{4jV_0^2}{\pi^2a\omega\mu}\frac{\left(\cos\frac{\pi l}{a}-\cos kl\right)^2}{\left\{1-\left(\frac{\lambda}{2a}\right)^2\right\}}\cdot\left[-\ln\frac{\pi w}{b}+\frac{\beta^2b^2}{2\pi^2}\sum_1^{\infty}\frac{1}{n^3}\right]. \quad (16a)$$

For $m>1$

$$\langle a,a\rangle_2=\frac{4jV_0^2}{\pi^2a\omega\mu}\sum_{m=3,5,7,\cdots}^{\infty}\frac{\left(\cos\frac{m\pi l}{a}-\cos kl\right)^2}{1-\left(\frac{m\lambda}{2a}\right)^2}k_0\left(k_m\frac{b}{2}\right). \quad (16b)$$

The total self-reaction in guide II is then given by the sum of (16a) and (16b).

For calculating the required susceptance, the discontinuity in the modal voltage due to dominant mode is given by [9]

$$V=V_0\sqrt{\frac{2}{ab}}\left[\frac{2\left(\cos\frac{\pi l}{a}-\cos kl\right)}{k\left\{1-\left(\frac{\lambda}{2a}\right)^2\right\}}\right]. \quad (17)$$

Hence, the required normalized susceptance can now be evaluated from (1), (2), (4), (5) and (14)–(17) as

$$\bar{B}=\frac{B}{y_0}=\frac{kb}{\pi^2}\left[\left\{1-\left(\frac{\lambda}{2a}\right)^2\right\}\cdot\left\{-\ln 2\sin\frac{\pi w}{2b}-\ln\frac{\pi w}{b}+\frac{\beta^2b^2}{\pi^2}\sum_1^{\infty}\frac{1}{n^3}\right\}-\sum_{3,5,7,\cdots}^{\infty}\left\{\frac{\cos\frac{m\pi l}{a}-\cos kl}{\cos\frac{\pi l}{a}-\cos kl}\right\}^2\frac{\left\{1-\left(\frac{\lambda}{2a}\right)^2\right\}^2}{\left(\frac{m\lambda}{2a}\right)^2-1}\cdot\left\{k_0(k_mw)+k_0\left(k_m\frac{b}{2}\right)\right\}\right]. \quad (18)$$

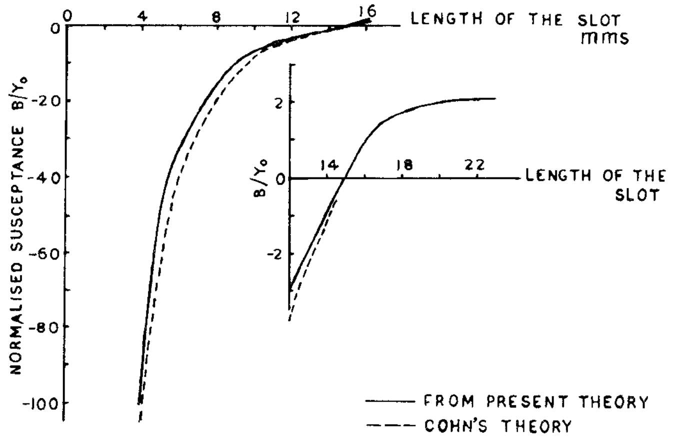

Fig. 3. Variation of normalized susceptance versus slot length for $\lambda=3.2$ cm. ——, from present theory (18); ----, from Cohn's theory.

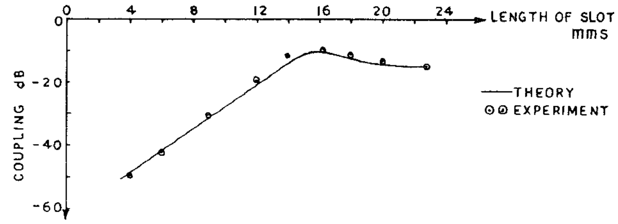

Fig. 4. Variation of coupling versus slot length ($\lambda=3.2$ cm).

Variation of normalized susceptance as a function of slot length in the range $4<2l<a$ (dimensions in millimeters) is computed from (18) for $\lambda=3.2$ cm, slot width $2w=1$ mm, and is presented in Fig. 3. Susceptance is negative (capacitive) for small length of the slot and passes through zero at $2l\approx 15$ mm. For $2l>15$ mm susceptance is positive (inductive). For smaller slot lengths, a larger susceptance is to be expected with thin rectangular slots, and this is confirmed in the graph. For length of the slot equal to the broad dimension of guide I ($2l=a$) the formula given by Levinson and Fredberg gives $\bar{B}=2.00$ and that of Marcuvitz gives $\bar{B}=2.0568$, which compares well with the value of $\bar{B}=2.059$ computed from (18).

For the sake of comparison, normalized susceptance is also calculated using the graph of magnetic polarizability given by Cohn [3] combined with his theory for long narrow slots [2] and the results[1] are presented in Fig. 3. Since the experimental results on magnetic polarizability are available for $0.07<w/l<1$, for a slot width of 1 mm, the comparison with Cohn's theory is limited to a slot length up to 14 mm.

III. Coupling Due to the Slot

The equivalent circuit of the junction at the reference plane $T-T$ is shown in Fig. 2. The ratio of the total modal voltage at port 3 to that at port 1 is given by

$$\left(\frac{V_3}{V_1}\right)_{d_B}=20\log\frac{1/\bar{B}}{|2+3j/\bar{B}|}+8.686-\alpha t. \qquad (19)$$

The first term of (19) is the coupling coefficient between port 1 and 3. The second term is the correction due to finite wall thickness which accounts for the attenuation of the principal mode in a waveguide whose cross section is the same as that of the aperture and whose length is equal to the wall thickness [2] t. The attenuation constant α is given by

$$\alpha=\sqrt{\left(\frac{\pi}{2l}\right)^2-k^2}\ .$$

The modal voltage V_3 is related to the incident and reflected wave amplitudes V_3^+ and V_3^- at port 3 by the relation

$$V_3=V_3^++V_3^-.$$

Similarly

$$V_1=V_1^++V_1^-.$$

Since port 3 is matched, $V_3^-=0$. Therefore,

$$\left(\frac{V_3}{V_1}\right)^2=\left(\frac{V_3^+}{V_1^+}\right)^2\left[\frac{1}{1+\left(\frac{V_1^-}{V_1^+}\right)^2}\right]=\left(\frac{V_3^+}{V_1^+}\right)^2\left(\frac{1}{1+|\Gamma|^2}\right). \qquad (20)$$

In order to determine coupling experimentally, the incident power P_1 is first measured by removing the tee junction and terminating the guide in a matched load. The tee junction is then inserted between the matched load and the guide. With port 3 terminated in a matched load, power P_3 in port 3 and VSWR in port 1 are measured. The ratio

$$\frac{P_3}{P_1}=\left(\frac{V_3^+}{V_1^+}\right)^2.$$

From the measured input VSWR, the magnitude of reflection coefficient $|\Gamma|$ is determined. From these measurements $(V_3/V_1)^2$ is calculated from (20).

The theoretical and experimental results on variation of coupling V_3/V_1, as a function of slot length (slot width = 1 mm, wall thickness $t=1.58$ mm) are presented in Fig. 4. There is a fairly good agreement between the theoretical and experimental results.

IV. Impedance Loading on Primary Guide

A knowledge of impedance loading on primary guide I is important for the design of cascaded sections of such tee junctions using loaded line analysis [7]. From the equivalent circuit shown in Fig. 2, we can write the expression for normalized impedance presented to guide I in terms of equivalent circuit parameter as follows:

$$\bar{Z}_{\text{in}}=\left(1+\frac{1}{1+\bar{B}^2}\right)+\frac{j\bar{B}}{1+\bar{B}^2}. \qquad (21)$$

[1]The fact that the proposed theory agrees closely with the results obtained by Cohn was brought to the attention of the authors by an anonymous reviewer, for which the authors are very grateful.

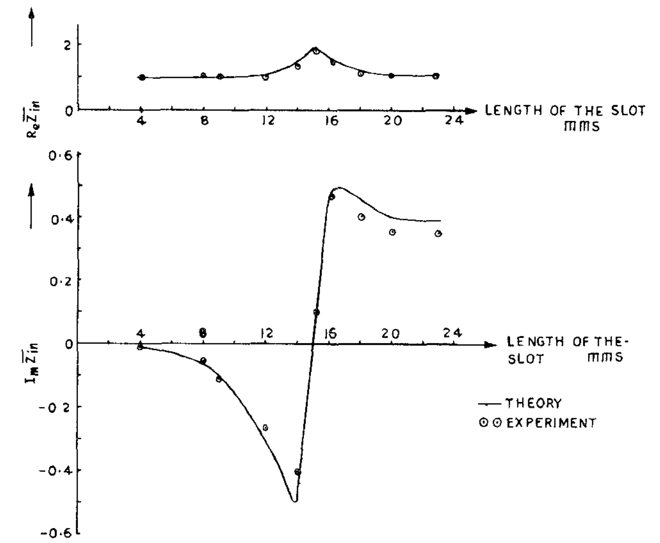

Fig. 5. Variation or real and imaginary parts of input impedance versus slot length for $\lambda = 3.2$ cm.

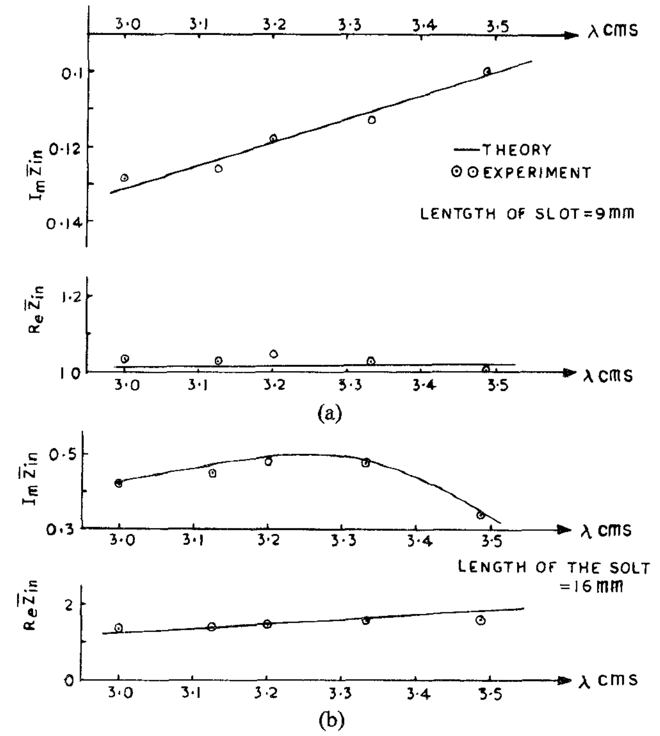

Fig. 6. Variation of real and imaginary parts of input impedance versus wavelength. (a) Slot length = 9 mm. (b) Slot length = 16 mm.

Input impedance was computed as a function of slot length for $\lambda = 3.2$ cm. Fig. 5 shows the variation of real and imaginary parts of $\bar{Z}_{in}$ with slot length. Experimental results are also shown for comparison. Variation of impedance loading as a function of frequency was also determined. Fig. 6 shows the real and imaginary parts of input impedance as a function of wavelength for slot lengths 9 and 16 mm. Experimental results presented in the same figure for the sake of comparison show good agreement with the theory.

V. Conclusion

Theoretical results for the equivalent circuit parameter show a close agreement with those obtained from Cohn's theory. Variation of coupling and impedance loading calculated from (19) and (21), respectively, is in good agreement with the experimental results. For the particular case of a slot of length equal to the broad dimension of the rectangular waveguide, the results obtained from Marcuvitz's and Levinson and Fredberg's formula compare well with the corresponding values obtained from (18).

The present theory based on a modal representation can be used for the analysis of electromagnetic coupling through apertures in the broad wall of any two arbitrary waveguides.

Acknowledgment

The authors would like to thank Prof. G. S. Sanyal and Prof. J. Das for their kind interest in the work. The valuable suggestions by Prof. V. U. Reddy are also acknowledged. The authors wish to thank the IEEE reviewers for their helpful comments.

References

[1] H. A. Bethe, "Theory of diffraction by small holes," *Phys. Rev.*, vol. 66, p. 163, 1944.
[2] S. B. Cohn, "Microwave coupling by large apertures," *Proc. IRC*, vol. 40, no. 6, pp. 696–699, June 1952.
[3] ——, "Determination of aperture parameters by electrolytic tank measurements," *Proc. IRE*, vol. 39, no. 11, pp. 1416–1421, Nov. 1951.
[4] N. Marcuvitz, *Waveguide Handbook*, R. L. Series. New York: McGraw-Hill, 1951, vol. 10, p. 374.
[5] I. B. Levinson and P. S. Fredberg, "Slot couplers of rectangular one mode waveguide—Equivalent circuits and lumped parameters," *Radio Eng. Electron. Phys.*, no. 1, pp. 717–724, 1966.
[6] ——, "Slot couplers of rectangular one mode waveguide—Numerical results," *Radio Eng. Electron. Phys.*, no. 6, p. 937, 1966.
[7] G. A. Yevastropov and S. A. Tsarapkin, "Investigations of slotted waveguide antennas with identical resonant radiators," *Radio Eng. Electron. Phys.*, vol. II, p. 1429, 1965.
[8] R. F. Harrington, *Time Harmonic Electromagnetic Fields*. New York: McGraw-Hill, 1961, chs. 3 and 8.
[9] N. Marcuvitz and J. Schwinger, "On the representation of electric and magnetic fields produced by currents and discontinuities in waveguides," *J. Appl. Phys.*, vol. 22, pp. 806–819, June 1951.
[10] G. Markov, *Antennas*. Moscow, USSR: Progress Publishers, 1965, ch. 7, p. 196.
[11] R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960, pp. 267–270.